

```
> Val_Propre:=(Matrice)-
>evalf(eigenvalues(inverse(multiply(transpose(Matrice),Matrice))));
Val_Propre := Matrice → evalf(eigenvalues(inverse(multiply(transpose(Matrice), Matrice))))
```

```
> Fct_Trace:=(Matrice)-
>trace(inverse(multiply(transpose(Matrice),Matrice)));
Fct_Trace := Matrice → trace(inverse(multiply(transpose(Matrice), Matrice)))
```

```
> Fct_Det:=(Matrice)-
>det(inverse(multiply(transpose(Matrice),Matrice)));
Fct_Det := Matrice → det(inverse(multiply(transpose(Matrice), Matrice)))
```

```
> Aeff:=(Matrice,n,p)->evalf(100*(1/(n*Fct_Trace(Matrice)/p)));
```

$$A_{eff} := (Matrice, n, p) \rightarrow \text{evalf}\left(\frac{100 p}{n Fct_Trace(Matrice)}\right)$$

```
> Deff:=(Matrice,n,p)->evalf(100*(1/(n*Fct_Det(Matrice)^(1/p))));
```

$$Deff := (Matrice, n, p) \rightarrow \text{evalf}\left(\frac{100}{n Fct_Det(Matrice)^{\left(\frac{1}{p}\right)}}\right)$$

```
> Poids:=matrix(4,1,[y1,y2,y3,y4]);
```

$$Poids := \begin{bmatrix} y1 \\ y2 \\ y3 \\ y4 \end{bmatrix}$$

```
> Plan1:=matrix(4,4,[[1,0,0,0],[1,1,0,0],[1,0,1,0],[1,0,0,1]]);
```

$$Plan1 := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$

```
> multiply(inverse(Plan1),Poids);
```

$$\begin{bmatrix} y1 \\ -y1+y2 \\ -y1+y3 \\ -y1+y4 \end{bmatrix}$$

```
> Plan2:=matrix(4,4,[[1,0,0,0],[1,1,1,0],[1,1,0,1],[1,0,1,1]]);
```

$$\text{Plan2} := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix}$$

```
> multiply(inverse(Plan2),Poids);
```

$$\begin{bmatrix} y1 \\ -\frac{1}{2}y1 + \frac{1}{2}y2 + \frac{1}{2}y3 - \frac{1}{2}y4 \\ -\frac{1}{2}y1 + \frac{1}{2}y2 - \frac{1}{2}y3 + \frac{1}{2}y4 \\ -\frac{1}{2}y1 - \frac{1}{2}y2 + \frac{1}{2}y3 + \frac{1}{2}y4 \end{bmatrix}$$

```
> Plan3:=matrix(4,4,[[1,1,1,1],[1,-1,-1,1],[1,1,-1,-1],[1,-1,1,-1]]);
```

$$\text{Plan3} := \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & -1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \end{bmatrix}$$

```
> multiply(inverse(Plan3),Poids);
```

$$\begin{bmatrix} \frac{1}{4}y^1 + \frac{1}{4}y^2 + \frac{1}{4}y^3 + \frac{1}{4}y^4 \\ \frac{1}{4}y^1 - \frac{1}{4}y^2 + \frac{1}{4}y^3 - \frac{1}{4}y^4 \\ \frac{1}{4}y^1 - \frac{1}{4}y^2 - \frac{1}{4}y^3 + \frac{1}{4}y^4 \\ \frac{1}{4}y^1 + \frac{1}{4}y^2 - \frac{1}{4}y^3 - \frac{1}{4}y^4 \end{bmatrix}$$

```
> Fct_Trace(Plan1);Fct_Det(Plan1);Val_Propre(Plan1);  
7 1 4.791287848, 0.208712152, 1., 1.
```

```
> Fct_Trace(Plan2);Fct_Det(Plan2);Val_Propre(Plan2);  
 $\frac{1}{4}$   
4 4 1.866025404, 0.1339745960, 1., 1.
```

```
> Fct_Trace(Plan3);Fct_Det(Plan3);Val_Propre(Plan3);  
 $\frac{1}{1256}$   
1 256 0.2500000000, 0.2500000000, 0.2500000000, 0.2500000000
```

```
> Aeff(Plan1,4,4);Aeff(Plan2,4,4);Aeff(Plan3,4,4);  
14.28571429 25. 100.
```

```
> Deff(Plan1,4,4);Deff(Plan2,4,4);Deff(Plan3,4,4);  
25. 35.35533905 100.0000000
```

```
> PlanA:=matrix(16,4,[[1,1,-1,1],[1,0.33,0,-1],[1,-1,-1,-1],[1,-0.33,1,0],[0.33,1,0,0],[0.33,0.33,-1,0],[0.33,-1,-1,-1],[0.33,-0.33,1,1],[-1,1,-1,-1],[-1,0.33,1,-1],[-1,-1,0,1],[-1,-.33,-1,0],[-0.33,1,1,-1],[-0.33,0.33,-1,1],[-0.33,-1,1,0],[-0.33,-0.33,0,-1]]);
```

```
PlanA :=
```

| | | | |
|-------|-------|----|----|
| 1 | 1 | -1 | 1 |
| 1 | 0.33 | 0 | -1 |
| 1 | -1 | -1 | -1 |
| 1 | -0.33 | 1 | 0 |
| 0.33 | 1 | 0 | 0 |
| 0.33 | 0.33 | -1 | 0 |
| 0.33 | -1 | -1 | -1 |
| 0.33 | -0.33 | 1 | 1 |
| -1 | 1 | -1 | -1 |
| -1 | 0.33 | 1 | -1 |
| -1 | -1 | 0 | 1 |
| -1 | -0.33 | -1 | 0 |
| -0.33 | 1 | 1 | -1 |
| -0.33 | 0.33 | -1 | 1 |
| -0.33 | -1 | 1 | 0 |
| -0.33 | -0.33 | 0 | -1 |

```
> Fct_Trace(PlanA);Fct_Det(PlanA);Val_Propre(PlanA);
```

```
0.4017643147 0.00009727484059  
0.08158411517, 0.09010781479, 0.1139811121, 0.1160912726
```

```
> Aeff(PlanA,16,4);Deff(PlanA,16,4);
```

```
62.22553644 62.93321019
```

```
> PlanB:=matrix(16,4,[[ -1,-0.33,-1,-1],[ -0.33,-1,0,-1],[0.33,-1,-1,1],[0.33,1,1,-1],[1,-1,1,0],[1,-0.33,0,1],[0.33,-0.33,-1,0],[ -1,0.33,1,1],[ -1,1,0,0],[1,1,-1,-1],[ -0.33,1,-1,1],[ -0.33,0.33,-1,0],[1,0.33,-1,-1],[ -0.33,-0.33,1,-1],[ -1,-1,-1,-1],[0.33,0.33,0,-1]]);
```

```
PlanB :=
```

| | | | |
|-------|-------|----|----|
| -1 | -0.33 | -1 | -1 |
| -0.33 | -1 | 0 | -1 |
| 0.33 | -1 | -1 | 1 |
| 0.33 | 1 | 1 | -1 |
| 1 | -1 | 1 | 0 |
| 1 | -0.33 | 0 | 1 |
| 0.33 | -0.33 | -1 | 0 |
| -1 | 0.33 | 1 | 1 |
| -1 | 1 | 0 | 0 |
| 1 | 1 | -1 | -1 |
| -0.33 | 1 | -1 | 1 |
| -0.33 | 0.33 | -1 | 0 |
| 1 | 0.33 | -1 | -1 |
| -0.33 | -0.33 | 1 | -1 |
| -1 | -1 | -1 | -1 |
| 0.33 | 0.33 | 0 | -1 |

```
> Fct_Trace(PlanB);Fct_Det(PlanB);Val_Propre(PlanB);
0.3932808106 0.00008885854989
0.07692307693, 0.09090909091, 0.1127243214, 0.1127243214
> Aeff(PlanB,16,4);Deff(PlanB,16,4);
63.56781040 64.37321682
```